

Let  $P$  be the point  $(-3, -1, 5)$ . Let  $Q$  be the point  $(-7, 1, -1)$ .

SCORE: \_\_\_\_ / 65 PTS

Let  $R$  be the point such that  $\overrightarrow{PR} = \langle 3, 2, 1 \rangle$ .

[a] Find the co-ordinates of  $R$ .

$$(-3+3, -1+2, 5+1) = (0, 1, 6)$$

[b] In the triangle  $\Delta PQR$ , find the measure of angle  $\angle RPQ$ .

$$\overrightarrow{PQ} = \langle -7 - (-3), 1 - (-1), -1 - 5 \rangle = \langle -4, 2, -6 \rangle$$

$$\cos^{-1} \frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\|\overrightarrow{PQ}\| \|\overrightarrow{PR}\|} = \cos^{-1} \frac{-12 + 4 - 6}{\sqrt{56} \sqrt{14}} = \cos^{-1} \frac{-14}{2(4)} = \cos^{-1} -\frac{1}{2} = \frac{2\pi}{3}$$

[c] Write  $\overrightarrow{PR}$  as the sum of two orthogonal vectors, one of which is the projection of  $\overrightarrow{PR}$  onto  $\overrightarrow{PQ}$ .

$$\frac{\overrightarrow{PQ} \cdot \overrightarrow{PR}}{\overrightarrow{PQ} \cdot \overrightarrow{PQ}} \overrightarrow{PQ} = \frac{-14}{56} \langle -4, 2, -6 \rangle = -\frac{1}{4} \langle -4, 2, -6 \rangle = \langle 1, -\frac{1}{2}, \frac{3}{2} \rangle$$

$$\langle 3, 2, 1 \rangle = \langle 1, -\frac{1}{2}, \frac{3}{2} \rangle + \langle 2, \frac{5}{2}, -\frac{1}{2} \rangle$$

[d] Find the area of the triangle  $\Delta PQR$ .

$$\begin{aligned} \overrightarrow{PQ} \times \overrightarrow{PR} &= \langle -4, 2, -6 \rangle \times \langle 3, 2, 1 \rangle \\ &= \langle 2 - 12, -(-4 - 18), -8 - 6 \rangle \\ &= \langle 14, -14, -14 \rangle \end{aligned}$$

$$\frac{1}{2} \|\overrightarrow{PQ} \times \overrightarrow{PR}\| = \frac{1}{2} (14) \|\langle 1, -1, -1 \rangle\| = 7\sqrt{3}$$

[e] Find the equation of the plane passing through  $P, Q$  and  $R$ . Write your final answer in general form  $Ax + By + Cz + D = 0$ .

$$14(x+3) - 14(y+1) - 14(z-5) = 0$$

$$\text{or } (x+3) - (y+1) - (z-5) = 0$$

$$x - y - z + 7 = 0$$

Let  $\vec{b} = 4\vec{i} - \vec{j} - 8\vec{k}$  and  $\vec{c} = -\vec{j} - 2\vec{k}$ .

SCORE: \_\_\_\_ / 55 PTS

Let  $L$  be the point  $(-3, -5, 2)$ .

[a] Write  $3\vec{c} - 2\vec{b}$  in component form.

$$3\langle 0, -1, -2 \rangle - 2\langle 4, -1, -8 \rangle = \langle 0, -3, -6 \rangle - \langle 8, -2, -16 \rangle \\ = \langle -8, -1, 10 \rangle$$

[b] Find a vector of magnitude 2 in the same direction as  $\vec{b}$ .

$$2\left(\frac{1}{\|\vec{b}\|}\right)\vec{b} = 2\left(\frac{1}{\sqrt{81}}\right)\langle 4, -1, -8 \rangle = \frac{2}{9}\langle 4, -1, -8 \rangle \\ = \left\langle \frac{8}{9}, -\frac{2}{9}, -\frac{16}{9} \right\rangle$$

[c] Find a unit vector perpendicular to both  $\vec{b}$  and  $\vec{c}$ .

$$\vec{b} \times \vec{c} = \langle 4, -1, -8 \rangle \times \langle 0, -1, -2 \rangle \\ = \langle 2 - 8, 0 - -8, -4 - 0 \rangle \\ = \langle -6, 8, -4 \rangle \\ \frac{1}{\|\vec{b} \times \vec{c}\|} (\vec{b} \times \vec{c}) = \frac{1}{\sqrt{116}} \langle -6, 8, -4 \rangle = \frac{1}{2\sqrt{29}} \langle -6, 8, -4 \rangle \\ = \left\langle \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{-2}{\sqrt{29}} \right\rangle$$

[d] Find the equation of the plane parallel to both  $\vec{b}$  and  $\vec{c}$ , and passing through point  $L$ .

Write your final answer in general form  $Ax + By + Cz + D = 0$ .

$$-6(x+3) + 8(y+5) - 4(z-2) = 0 \\ \text{OR } 3(x+3) - 4(y+5) + 2(z-2) = 0 \\ 3x - 4y + 2z - 15 = 0$$

[e] Let  $\ell_1$  be the line with parametric equation  $x = 6 - 4t$ ,  $y = t + 9$ ,  $z = 2t - 7$ .

Find the symmetric equation of the line parallel to  $\ell_1$  and passing through  $L$ .

$$\frac{x+3}{-4} = \frac{y+5}{1} = \frac{z-2}{2} \\ \text{OR } -\frac{x+3}{4} = y+5 = \frac{z-2}{2}$$

Let  $\vec{r}$  be the vector with magnitude 4 and direction angle  $\frac{\pi}{6}$ . Let  $\vec{s} = \langle -4\sqrt{3}, 4 \rangle$ .

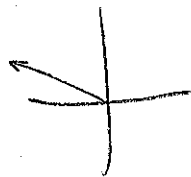
SCORE: \_\_\_\_ / 20 PTS

[a] Find the component form of  $\vec{r}$ .

$$\langle 4 \cos \frac{\pi}{6}, 4 \sin \frac{\pi}{6} \rangle = \langle 2\sqrt{3}, 2 \rangle$$

[b] Find the direction angle of  $\vec{s}$ .

$$\tan^{-1} \frac{4}{-4\sqrt{3}} = \tan^{-1} -\frac{\sqrt{3}}{3} = -\frac{\pi}{6} \text{ in } Q_4$$


$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

[c] If  $\vec{r}$  represents a force, and  $\vec{s}$  is the movement of an object that the force is applied to, find the work done.

$$\langle 2\sqrt{3}, 2 \rangle \cdot \langle -4\sqrt{3}, 4 \rangle = -8(3) + 8 = -16$$

If  $\langle 6, a, -12 \rangle$  is parallel to  $\langle b, -2, 9 \rangle$ , find the values of  $a$  and  $b$ .

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$$\langle 6, a, -12 \rangle = k \langle b, -2, 9 \rangle$$

$$6 = kb$$

$$a = -2k$$

$$-12 = 9k \rightarrow k = -\frac{4}{3}$$

$$a = -2\left(-\frac{4}{3}\right) = \frac{8}{3}$$

$$6 = -\frac{4}{3}b \rightarrow b = -\frac{9}{2}$$